St Peter's C.E. Academy

÷ x + −

Our Approach to Calculations

This booklet outlines a variety of written calculations used for the four number operations. These written methods are based around a sound understanding of times tables along with being able to rapidly recall associated division facts. A good understanding of number bonds to 10, 20 and 100, along with the number system and place value are also essential. Children will use equipment and pictures to support their understanding. Children will need fluent number skills to reason and problem solve.

Throughout the booklet the strategies for each operation have been organised so that children can build upon these methods. It is vitally important that the children understand their chosen method and are capable of explaining this to others. When a new strategy is introduced, previous learning will be built upon to develop progression across the four number operations.

The children are expected to choose the most efficient written method for each of the number operations that they can explain and understand securely.

ADDITION

Mental Skills.

* Count on in ones and tens.
* Know number bonds to 10, 20 and 100.
* Recognise the size and position of numbers.
* Add multiples of 10 and 100 to any number.
* Partition and recombine numbers.

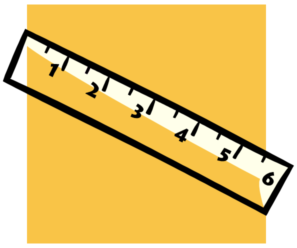
add **count on**

addition **plus**

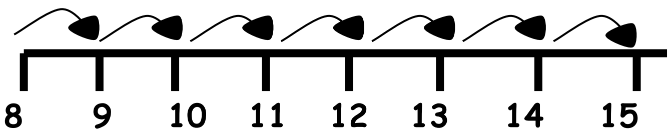
more **sum** **total**

altogether **increase**

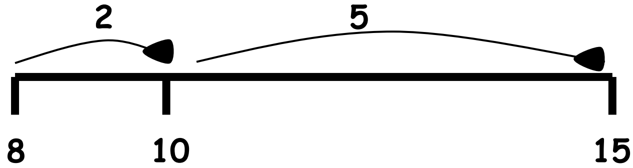
Count on using a number line or ruler.



8 + 7 = 15

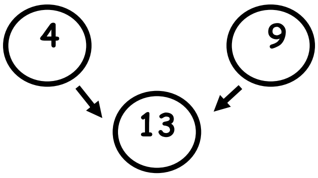


OR



Part Whole model

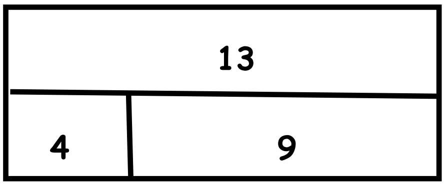
Shows the relationship between the whole number and the component parts.



4 + 9 = 13

Bar model

A pictorial representation of a number in the form of a bar. This can be used to problem solve with larger numbers.



4 + 9 = 13

Add numbers in a different order to make the calculation easier.

8 + 7 + 2 + 5 + 3

is the same as

8 + 2 + 7 + 3 + 5

10 + 10 + 5 = 25

LOOK FOR NUMBER BONDS!

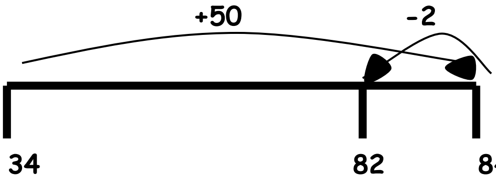
37 + 48 + 23

is the same as

37 + 23 + 48

= 60 + 48

= 108

<u>Near Doubles.</u>	8 + 9 = 17 **is the same as 1 more than double 8** (16 + 1 = 17) **or 1 less then double 9** (18 - 1 = 17) **37 + 36 = 73** **is the same as 1 more than double 36** (72 + 1 = 73) **or 1 less then double 37** (74 - 1 = 73)
<u>Near Multiple of 10 and Adjust.</u>	**34 + 48 = 82** **is the same as 34 add 50 and subtract 2**  **64 + 71 = 135 is the same as 64 plus 70 plus 1 more**
<u>Partitioning.</u> **TU + TU** **Mental strategy with jottings.**	**TU + TU** **86 + 37 = 123** **86 + 37 = 123** **(80 + 30) + (6 + 7) = 123** $\downarrow$ $\downarrow$ **110 + 13 = 123** **<u>OR</u>** $\begin{array}{r} 80 + 6 \\ 30 + 7 \\ \hline 110 + 13 = 123 \\ \hline \end{array}$
<u>Expanded Column Method.</u> Add the **<u>units</u>** first You should be able to explain clearly what you are doing with understanding of place value	$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \\ \hline \end{array}$ $\begin{array}{r} 648 \\ +\ 286 \\ \hline 14 \\ 120 \\ 800 \\ \hline 934 \\ \hline \end{array}$
<u>Column Method with Carrying</u> Carry digits are recorded **<u>below the line</u>**	$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline 1 \end{array}$ $\begin{array}{r} 648 \\ +286 \\ \hline 934 \\ \hline 11 \end{array}$ $\begin{array}{r} 68.74 \\ +26.6 \\ \hline 95.34 \\ \hline 1 \end{array}$ Extend understanding into decimals and in context of money and measures.

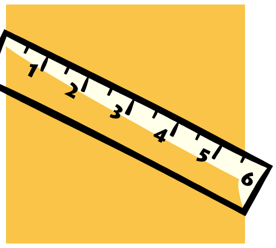
SUBTRACTION

Mental Skills

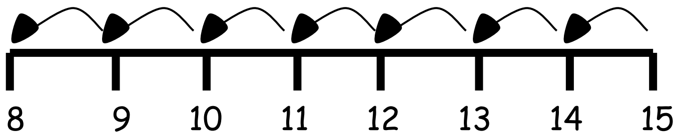
* Recognise the size and position of numbers.
* Count back in ones and tens.
* Subtract multiples of 10 and 100 from any number.

takeaway **subtract**
reduce **less**
count back **minus**
fewer **difference between**
decrease **exchanging**

Count back using a number line or ruler.



15 - 7 = 8



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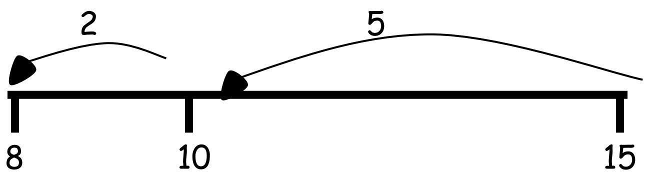
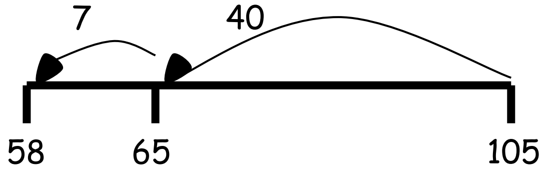
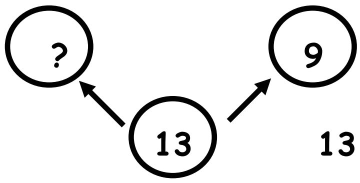


105 - 47 = 58

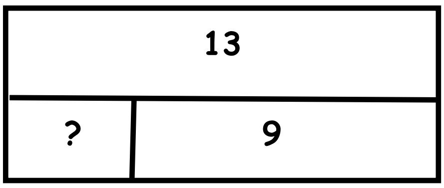


Part Whole model
Shows the relationship between the whole number and the component parts.



13 - 9 = ?

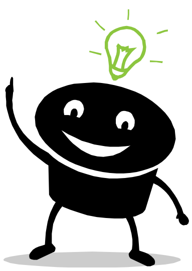
Bar model
A pictorial representation of a number in the form of a bar. This can be used to problem solve with larger numbers.



13 - ? = 9

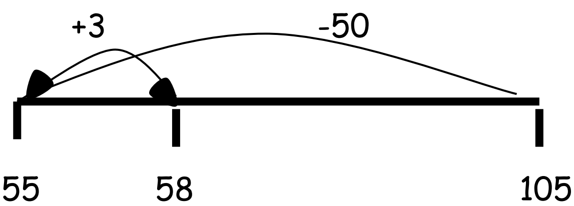
Near Multiple of Ten and Adjust.

This is a good mental strategy.



105 - 47 = 58

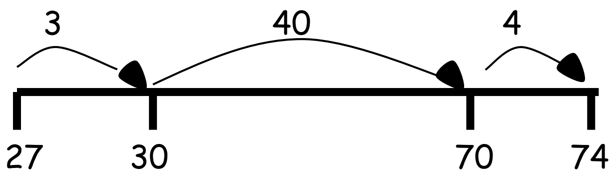
Same as subtract 50 and then add back 3



Counting On Using a Number Line
OR Set Out Vertically.

This method can be used with decimals but it becomes less efficient with decimals and bigger numbers.

74 - 27 = 47



Leading to

$$\begin{array}{rl} 74 & \\ -27 & \\ \hline +\ 3 & 30 \\ +\ 40 & 70 \\ +\ 4 & 74 \\ \hline =47 & \\ \hline \end{array} \qquad \begin{array}{rl} 74 & \\ -27 & \\ \hline +\ 3 & 30 \\ +\ 44 & 74 \\ \hline =47 & \\ \hline \end{array}$$

Column method without exchanging. (Also known as the Vertical method.)

$$\begin{array}{r} 96 \\ -\ 43 \\ \hline 53 \end{array}$$

Leading to

$$\begin{array}{r} 376 \\ -134 \\ \hline 242 \end{array}$$

Expanded Method Leading to Column Method.

TU - TU

The expanded method leads to the more compact method. Only move onto this method once you have secure mental skills and you have a clear understanding of when partitioning numbers is required.

74 - **27 = 47**

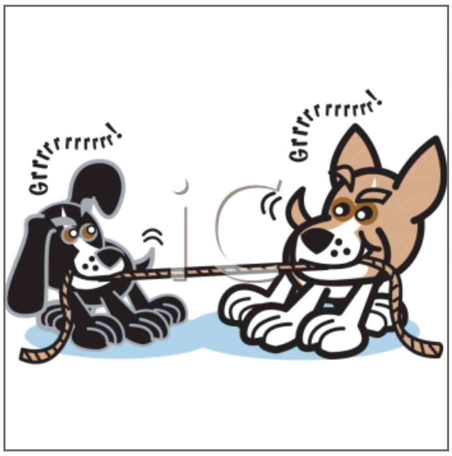
$$\begin{array}{rcl} \text{T} & & \text{U} \\ \boxed{60} & & \boxed{14} \\ \cancel{70} & + & \cancel{4} \\ -\ 20 & + & 7 \\ \hline 40 & + & 7 \\ \hline \end{array}$$

Leading to

$$\begin{array}{rcc} & \boxed{60} & \boxed{14} \\ & \cancel{7} & \cancel{4} \\ - & 2 & 7 \\ \hline & 4 & 7 \\ \hline \end{array}$$

Expanded Method Leading to Column Method.

HTU - HTU



Exchanging

Exchange from the next column along if you can't takeaway.

741 - 367 = 374

$$\boxed{130}$$

$$\boxed{600} \quad \boxed{\cancel{30}} \quad \boxed{11}$$

$$\cancel{700} + \cancel{40} + \cancel{1}$$

$$- \; 300 + 60 + 7$$

$$300 + 70 + 4$$

$$\boxed{13}$$

$$\boxed{6} \; \boxed{\cancel{3}} \; \boxed{11}$$

$$\cancel{7} \; \cancel{4} \; \cancel{1}$$

$$- \; 3 \; 6 \; 7$$

$$3 \; 7 \; 4$$

Are you choosing the best strategy for yourself?

Extend Column Method to Include Zero Values.

604 - 248 = 356

$$\boxed{90}$$

$$\boxed{500} \quad \boxed{\cancel{100}} \quad \boxed{14}$$

$$\cancel{600} + \cancel{0} + \cancel{4}$$

$$- \; 200 + 40 + 8$$

$$300 + 50 + 6$$

Leading to

↓

$$\boxed{9}$$

$$\boxed{5} \; \boxed{\cancel{10}} \; \boxed{14}$$

$$\cancel{6} \; \cancel{0} \; \cancel{4}$$

$$- \; 2 \; 4 \; 8$$

$$3 \; 5 \; 6$$

<u>Extended Column Method to Include Decimals and Zero Values.</u>

Line up the decimal points and place an extra zero if necessary.

90. 4 - 58. 73 = 31. 67

$$\begin{array}{ccccc}
 & \boxed{9} & & \boxed{13} & \\
\boxed{8} & \boxed{\cancel{10}} & & \boxed{\cancel{3}} & \boxed{10} \\
\cancel{9} & \cancel{0} & . & \cancel{4} & 0 \\
-\;5 & 8 & . & 7 & 3 \\
\hline
3 & 1 & . & 6 & 7 \\
\hline
\end{array}$$

Are you choosing the best strategy for yourself?

<u>Choose the best strategy for yourself.</u>

A column method may not always be the most efficient strategy!

What about 90. 4 - 29.9?

<u>Near multiple and adjust</u>

Subtract 30 = 60.4
Add back 0.1 = 60.5

What about using a column method 3564-1997?

Is this is the most efficient method?

Adjust and solve mentally!

<u>Near multiple and adjust</u>

Subtract 2000 = 1564
Add back 3 = 1567

MULTIPLICATION

Mental Skills

* Count on in different steps.
* Double and halve numbers.
* Recognise multiplication as repeated addition.
* Use known facts to derive associated numbers.
* Multiply by 10, 100 and 1000.
* Quick recall of multiplication facts up to 12x12.
* Multiplying by multiples of 10.
* Estimation.

multiplication times column

product double lots of row

groups of lots of multiply

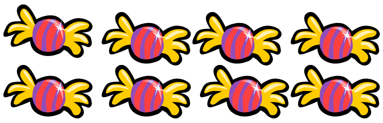
once twice three times

multiple repeated addition array

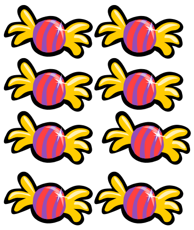
Count on in groups and record as arrays.

2 groups of 4 = 8 4 groups of 2 = 8

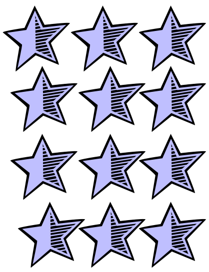
2 x 4 = 8



4 x 2 = 8



How many groups of 3 can be made from 12?
How many groups of 4 can be made from 12?



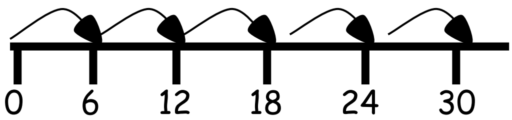
Apply times tables and associated facts to groupings

3 x 4 = 12
4 x 3 = 12

12 ÷ 4 = 3
12 ÷ 3 = 4

Count on in groups along a number line or ruler.

5 x 6 = 30



Doubling and Halving.

By doubling one number and halving the other you will create a question with the same answer.

5 x 16 = 80 **14 x 3 = 42** **15 x 8 = 120**
10 x 8 = 80 **7 x 6 = 42** **30 x 4 = 120**

Useful for finding factors of a number
1 x 40 = 40
2 x 20 = 40
4 x 10 = 40
5 x 8 = 40

Multiplying by 10,100 and 1000.

Multiplying	**Digits Move**
X 10	**1 place LEFT**
X 100	**2 places LEFT**
X 1000	**3 places LEFT**

25 x 10 = 250

Hundred 100	Ten 10	Unit 1
	2	5
2	5	0

25 x 100 = 2500

Thousand 1000	Hundred 100	Ten 10	Unit 1
		2	5
2	5	0	0

9. 52 x 10 = 95. 2

Ten 10	Unit 1		Tenth 0.1	Hundredth 0.01
	9	.	5	2
9	5	.	2	

9. 52 x 1000 = 9520

Thousand 1000	Hundred 100	Ten 10	Unit 1		Tenth 0.1	Hundredth 0.01
			9	.	5	2
9	5	2	0	.		

Partition Numbers.

34 x 6 = 204
(30 x 6) + (4 x 6)
180 + 24 = 204

Grid Multiplication.
TU x U

You can use the same strategy for
HTU x U

TU x U
34 x 6 = 204

X	**30**	**4**
6	**180**	**24**

180 + 24 = 204

Grid Multiplication.

TU x TU

76 x 49 = 3724

X	70	6
40	2800	240
9	630	54

Choose the easiest way to add them up?

$$\begin{array}{r} 3040 \\ +\ 684 \\ \hline 3724 \\ \hline \boxed{1} \end{array} \quad \text{OR} \quad \begin{array}{r} 3430 \\ +\ 294 \\ \hline 3724 \\ \hline \boxed{1} \end{array}$$

Expanded Vertical Multiplication.

TU x U
TU x TU
You can choose the same strategy for:

HTU x U
HTU x TU
ThHTU x TU

34 x 6 = 204

$$\begin{array}{r} 34 \\ \times\ \ 6 \\ \hline 24 \\ 180 \\ \hline 204 \\ \hline \boxed{1} \end{array}$$

76 x 49 = 3724

$$\begin{array}{r} 76 \\ \times\ 49 \\ \hline 54 \\ 630 \\ 240 \\ 2800 \\ \hline 3724 \\ \hline \boxed{1}\boxed{1} \end{array}$$

Compact Vertical Multiplication

Partial carrying of digits can be wrote on the doorstep.

$$\begin{array}{r} 34 \\ \times\ \ 6 \\ \hline 204 \\ \hline \boxed{2} \end{array} \qquad \begin{array}{r} 76 \\ \times\ 49 \\ \hline 684 \\ 3040 \\ \hline 3724 \\ \hline \boxed{1} \end{array}$$

Extend on

HTU x TU and ThHTU x TU

Estimation is a key process with all calculations but especially important when working with larger numbers and decimals.

274 x 32 = 8768

Estimate

300 x 30 = 9000

```
     2 7 4
 X     3 2
 ---------
     5 4 8
   8 2 2 0
 ---------
   8 7 6 8
 ---------
```

2959 x 28 = 82852

Estimate

3000 x 30 = 90000

```
    2 9 5 9
 X      2 8
 ----------
  2 3 6 7 2
  5 9 1 8 0
 ----------
  8 2 8 5 2
 ----------
```

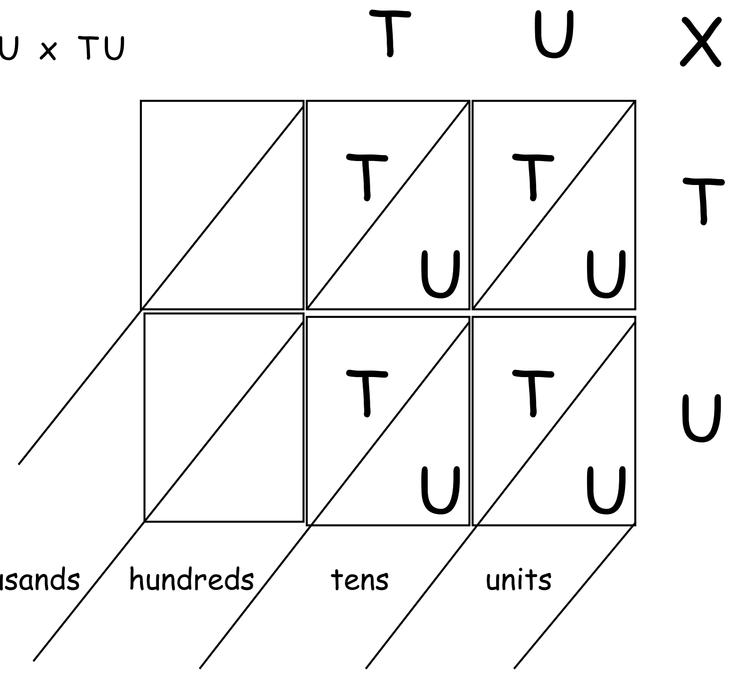
Gelosia Method

Set up the grid by marking its rows and columns with the numbers to be multiplied. Next, fill in the boxes with tens digits in the top triangles and units digits on the bottom.

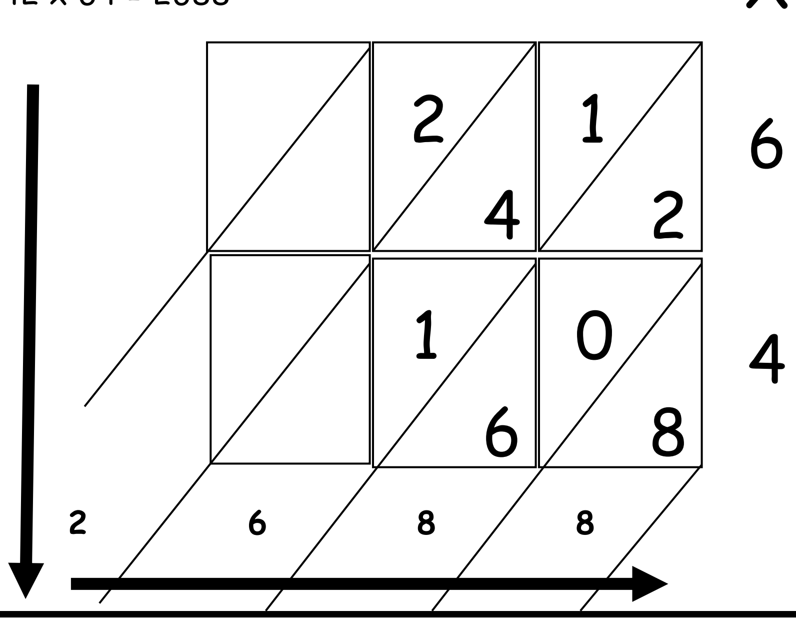
Read LEFT to RIGHT

(down the stairs, across the landing)

TU x TU



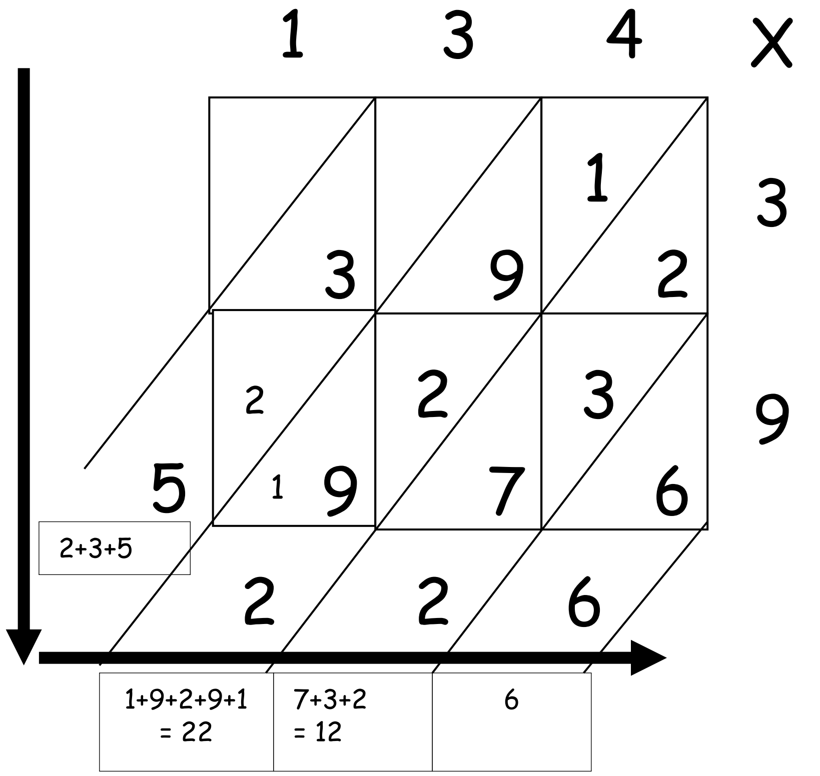
42 x 64 = 2688



Multiply HTU x TU with carrying.

This method also works for ThHTU x TU.

134 x 39 = 5226



Working with decimals.

Decimals

3 .4 x 7 .5 = 25.5

Estimate

3 x 8 = 24

Think of equivalent calculation

34 x 75 ÷ 100

Multiply digits together (ignore decimals)

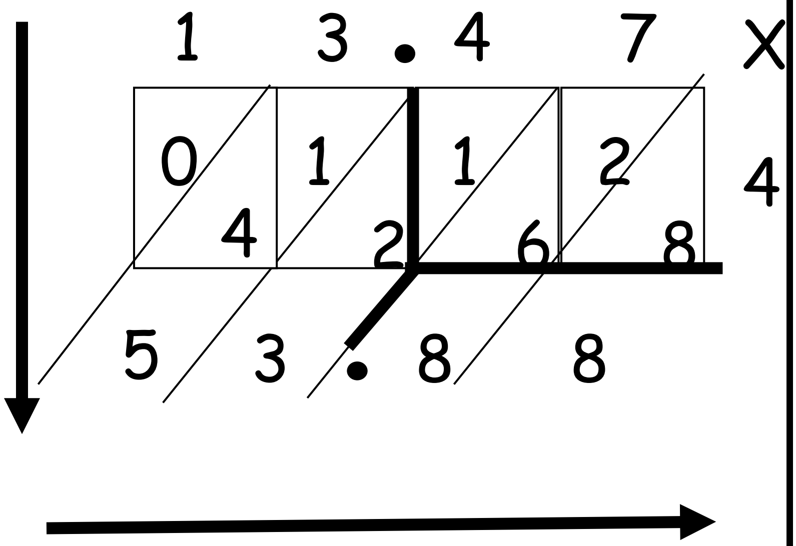
```
    3 4
  X 7 5
  -----
  1 7 0
2 3 8 0
-------
2 5 5 0
```

2550 ÷ 100 ⟶

Thousand 1000	Hundred 100	Ten 10	Unit 1		Tenth 0.1
2	5	5	0	.	
		2	5	.	5

<u>Gelosia Method to multiply decimals.</u>

Draw a line from each decimal point until it meets then extend diagonally left through the grid.



13.47 x 4 = 53.88

Which method is most efficient for what you are being asked to do?

Can you explain how to use this method to somebody else?

DIVISION

Mental Skills

* Count back in different steps.
* Double and half numbers.
* Recognise division as repeated subtraction.
* Quick recall of division facts that relate to multiplication facts up to 12 x 12.
* Use known facts to derive associated divisions.
* Divide by 10, 100 and 1000.
* Divide by multiples of 10.

divide **lots of** **half** **halve**

divisible **group** **groups of**

divide by **dividend**

quotient **remainder**

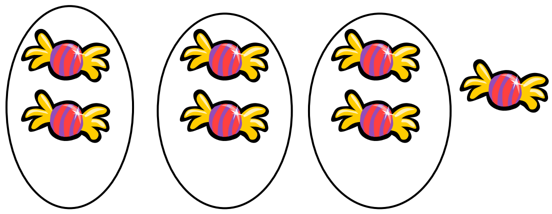
divisor **factor** **share** **halve**

Key Language $18 \div 3 = 6$

18 is the dividend, 3 is the divisor, 6 is the quotient

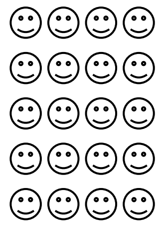
Practical examples of sharing including remainders.

I have 7 sweets to share between 3 of us, what shall I do?

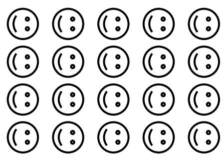


Understand grouping and be able to explain arrays.

(grouping)

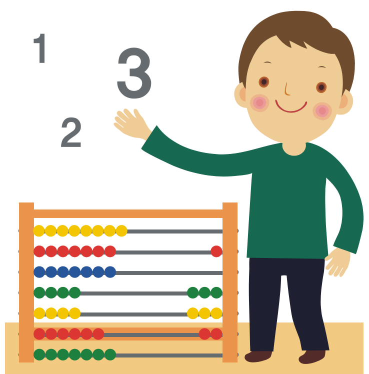


20 divided into groups of **4**
Gives **5** groups **(20 ÷ 4 = 5)**

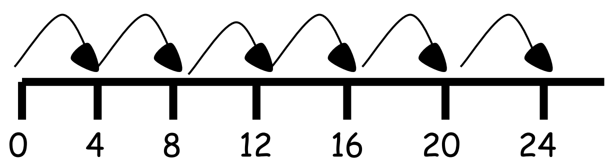


20 divided into groups of **5**
Gives 4 groups **(20 ÷ 5 = 4)**

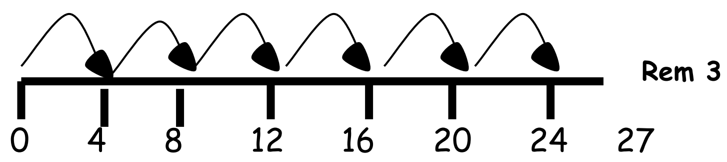
Use a number line to count on in groups.



24 ÷ 4 = 6



27 ÷ 4 = 6 remainder 3



Dividing by 10,100 and 1000.

Dividing	Digits Move
÷ 10	1 place RIGHT
÷ 100	2 places RIGHT
÷ 1000	3 places RIGHT

250 ÷ 10 = 25

Hundred 100	Ten 10	Unit 1
2	5	0
	2	5

2500 ÷ 100 = 25

Thousand 1000	Hundred 100	Ten 10	Unit 1
2	5	0	0
		2	5

95.2 ÷ 10 = 9.52

Ten 10	Unit 1		Tenth 0.1	Hundredth 0.01
9	5	.	2	
	9	.	5	2

360 ÷ 1000 = 0.36

Hundred 100	Ten 10	Unit 1		Tenth 0.1	Hundredth 0.01
3	6	0	.		
		0	.	3	6

Use known multiplication facts to work out associated divisions.

24 ÷ 4 = 6 use 4 x ? = 24

35 ÷ 7 = 5 use 7 x ? = 35

Mental division using Partitioning.

Multiples of 10

64 ÷ 4 = 16

(40 ÷ 4) + (24 ÷ 4)

10 + 6 = 16

102 ÷ 3 = 34

(90 ÷ 3) + (12 ÷ 3)

30 + 4 = 34

91 ÷ 7 = 13

(70 ÷ 7) + (21 ÷ 7)

10 + 3 = 13

196 ÷ 6 = 32 rem 4

(180 ÷ 6) + (12 ÷ 6)

30 + 2 = 32 rem 4

$= 32\frac{4}{6} = 32\frac{2}{3}$

Short Division.

Dividing by a single digit with carrying.

64 ÷ 4 = 16

91 ÷ 7 = 13

102 ÷ 3 = 34

$$\begin{array}{r} 1\ \ 6 \\ 4\,\overline{)\,6\ {}^{2}4} \end{array} \qquad \begin{array}{r} 1\ \ 3 \\ 7\,\overline{)\,9\ {}^{2}1} \end{array}$$

$$\begin{array}{r} 0\ \ 3\ \ 4 \\ 3\,\overline{)\,1\ {}^{1}0\ {}^{1}2} \end{array}$$

What to do with remainders - decimals or fraction?

196 ÷ 6 = 32 rem 4

$$\begin{array}{r} 0\ \ 3\ \ 2 \text{ rem } 4 \\ 6\,\overline{)\,1\ \boxed{{}^{1}}9\ \boxed{{}^{1}}6} \end{array} = 32\boxed{\tfrac{4}{6}} = 32\boxed{\tfrac{2}{3}}\ \boxed{\text{simplified}}$$

350 ÷ 8 = 43 rem 6

$$\begin{array}{r} 0\ \ 4\ \ 3 \text{ rem } 6 \\ 8\,\overline{)\,3\ \boxed{{}^{3}}5\ \boxed{{}^{3}}0} \end{array}$$

$$\begin{array}{r} 0\ \ 4\ \ 3\ .\ 7\ \ 5 \\ 8\,\overline{)\,3\ \boxed{{}^{3}}5\ \boxed{{}^{3}}0\ .\boxed{{}^{6}}0\ \boxed{{}^{4}}0} \end{array} = 43\boxed{\tfrac{6}{8}} = 43\boxed{\tfrac{3}{4}}$$

Chunking Down.

The key to efficiency with this strategy lies in the estimate that is made before the chunking starts.

e.g. for 196 ÷ 6
6 x 10 = 60 6 x 20 = 120
6 x 30 = 180 6 x 40 = 240
Therefore the answer lies between 30 and 40.

This method is based on subtracting multiples of the divisor. Initially children subtract several chunks but with practice they **should look for the biggest multiples** of the divisor to subtract.

102 ÷ 3 = 34

$$\begin{array}{rl} 3\ 4 & \\ 3\,\overline{)\,1\ 0\ 2} & \\ -9\ 0 & (\times\ 30) \\ \hline 1\ 2 & \\ -1\ 2 & (\times\ 4) \\ \hline 0 & 34 \end{array}$$

196 ÷ 6 = 32 rem 4 = 32 $\boxed{\tfrac{4}{6}}$ = 32 $\boxed{\tfrac{2}{3}}$

$$\begin{array}{rl} 3\ 2\ r\ 4 & \\ 6\,\overline{)\,1\ 9\ 6} & \\ -6\ 0 & (\times\ 10) \\ \hline 1\ 3\ 6 & \\ -6\ 0 & (\times\ 10) \\ \hline 7\ 6 & \\ -6\ 0 & (\times\ 10) \\ \hline 1\ 6 & \\ -1\ 2 & (\times\ 2) \\ \hline 4 & \end{array} \qquad \begin{array}{rl} 3\ 2\ r\ 4 & \\ 6\,\overline{)\,1\ 9\ 6} & \\ -1\ 8\ 0 & (\times\ 30) \\ \hline 1\ 6 & \\ -1\ 2 & (\times\ 2) \\ \hline 4 & \end{array}$$

Long Division.

Using factors to support long division.

1062 ÷ 18

9 and 2 are factors of 18.

$$\begin{array}{r} 0\,5\,3\,1 \\ 2\,)\,\overline{1\,{}^{1}0\,6\,2} \end{array}$$

$$\begin{array}{r} 5\;\;9 \\ 9\,)\,\overline{5\,{}^{5}3\,{}^{8}1} \end{array}$$

First complete 1062 ÷ 2 = 531
Next complete 531 ÷ 9 = 59
Therefore 1062 ÷ 18 = 59

Use factors to be more efficient. This method will only work if there are no remainders.

Long Division.

HTU ÷ TU and ThHTU ÷ TU

List the times tables you're working with to help you get started.

560 ÷ 24 = 23 rem 8 = 23 $\frac{8}{24}$ = 23 $\frac{1}{3}$

simplified

How many lots of 24 can we make from 560?

Estimate first.

24 x 10 = 240   24 x 20 = 480   24 x 30 = 720

So the answer will be between 20 and 30.

$$\begin{array}{rl} 2\,3\,\text{r}\,8 & \\ 24\,)\,\overline{5\,6\,0} & \\ -4\,8\,0 & (\times 20) \\ 8\,0 & \\ -7\,2 & (\times 3) \\ \hline 8 & \end{array}$$

24 Times Table	
1	24
2	48
3	72
4	96
5	120
6	144
7	168
8	192
9	216
10	240

BODMAS

It is common, when approaching a calculation in mathematics to ask yourself 'where do I start?' or 'what do I do first?'. There is a set order in which you should undertake the basic operations of arithmetic (adding, subtracting, multiplying and dividing). The acronym BODMAS helps you remember this order

What is BODMAS?

BODMAS stands for Brackets, Order, Divide, Multiply, Add, Subtract and reminds you in what sequence to carry out the operations for arithmetic.

(B)rackets ( ) **FIRST**

(O)rder $^2\sqrt{}$ (SQUARED OR SQUARE ROOTS ETC)

(D)ivision ÷ } **LEFT TO RIGHT**

(M)ultiplication X **LEFT TO RIGHT**

(A)ddition **+** } **LEFT TO RIGHT**

(S)ubtraction **-** **LEFT TO RIGHT**

When you see something like...

$7 + (6 \times 5^2 + 3)$

... what part should you calculate first?

Start at the left and go to the right?
Or go from right to left?

Calculate them in the wrong order, and you will get a wrong answer !

Do things in Brackets First. Example

✓ $6 \times \underline{(5 + 3)} = 6 \times \underline{8} = \mathbf{48}$

✗ $6 \times (5 + 3) = 30 + 3 = 33$ (wrong)

Then "Orders" (Powers, Roots) before Multiply, Divide, Add or Subtract. Example:

✓ $5 \times \underline{2^2} = 5 \times \underline{4} = \mathbf{20}$

✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Then Multiply or Divide before you Add or Subtract. Example:

✓ $2 + \underline{5 \times 3} = 2 + \underline{15} = \mathbf{17}$

✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right. Example:

✓ $30 \div 5 \times 3 = 6 \times 3 = \mathbf{18}$

✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

So ...... How Do I Remember It All Again... ? BODMAS !

B Brackets first
O Orders (ie Powers and Square Roots, etc.)
DM **D**ivision and **M**ultiplication (left-to-right)
AS **A**ddition and **S**ubtraction (left-to-right)

Divide and Multiply rank equally (and go left to right).
Add and Subtract rank equally (and go left to right)

After you have done "B" and "O", just go from left to right doing any "D" ***or*** "M" as you find them.
Then go from left to right doing any "A" ***or*** "S" as you find them.